DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE

DEPARTMENT OF MATHEMATICS

QUESTION BANK

MA1256 – DISCRETE MATHEMATICS

Year : III **Branch: CSE**

UNIT – I

PROPOSITIONAL CALCULUS

Part – A (2 Marks)

1. Write the negation of the following proposition. " To enter into the country you need a passport or a voter registration card".
2. How can this English sentence be translated into a logical expression?
   " You can access the Internet from campus only if you are computer science major or you are not freshman".
3. Makes a truth table for the statement $(p \wedge q) \vee (\sim p)$
4. State the truth table of "If tigers have wings then the earth travels round the sun".
5. Construct the truth table for $P \vee \leftarrow Q$
6. Construct the truth table for $P \wedge (P \vee Q)$
7. Construct the truth table for $(P \vee Q) \vee \leftarrow P$
8. Construct the truth table for (a) $\leftarrow(\leftarrow P \vee \leftarrow Q)$ (b) $\leftarrow(\leftarrow P \wedge \leftarrow Q)$
9. Construct the truth table for $(P \rightarrow Q) \wedge (Q \rightarrow P)$
10. Construct the truth table for $(Q \wedge (P \rightarrow Q)) \rightarrow P$
11. Construct the truth table for $\leftarrow(P \wedge Q) \leftrightarrow (\leftarrow P \vee \leftarrow Q)$
12. Construct the truth table for $\neg(P \vee Q) \rightarrow P$
13. n the converse and the contra positive of the implication "If it is a raining the I get wet".
14. Define contrapositive of a statement.
15. Using the truth table verify that the proposition $(P \wedge Q) \wedge \leftarrow(P \vee Q)$
16. Using truth table verify that the proposition $(P \wedge Q) \wedge \leftarrow(P \vee Q)$ is a contradiction.
17. Prove by truth tables that $\leftarrow(P \leftrightarrow Q) \Leftrightarrow (\leftarrow P \vee \leftarrow Q) \wedge (P \vee Q)$
18. Define the term logically equivalent.
19. Write the equivalent of the conditional p→q using disjunction ($\vee$).
20. Express the bi-conditional p↔q in any form using disjunction ($\vee$), conjunction ($\wedge$) and negation.

21. If P,Q and R are statement variable prove that
$P \wedge ((\leftarrow P \wedge Q) \vee (\leftarrow P \wedge \leftarrow Q)) \Rightarrow R$
22. Prove that whenever A$\wedge$ B$\Rightarrow$C, we also have A$\Rightarrow$ $(B \rightarrow C)$ and vic versa.
23. Show that {$\uparrow$ } is a minimal functionally complete set.
24. Express $P \uparrow Q$ interms of $\downarrow$ only.
25. Define the term NAND and NOR.
26. Define functionally complete set of connectives and given an example .
27. Show that $(\leftarrow P \wedge (\leftarrow Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ use only notation.
28. Show that $\leftarrow (P \wedge Q) \rightarrow (\leftarrow P \vee (\leftarrow P \vee Q)) \Leftrightarrow (\leftarrow P \vee Q)$
29. Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R \Leftrightarrow P \rightarrow (\leftarrow Q \vee R)$
30. Show that $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$
31. Show that $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$
32. Obtain PDNF for ~P$\vee$ Q
33. Define DNF.
34. Obtain disjunctive normal forms of $P \wedge (P \rightarrow Q)$
35. Define C.N.F.
36. Define P.D.N.F. (or) The sum of products normal form.
37. Define Minterms (or) Boolean conjunctions.
38. Define P.C.N.F.
39. Define Max terms (or) Boolean disjunction.
40. Define Inference theory.
41. Define valid arguments or valid conclusion.
42. State the rule of inference theory.
43. If the premises P,Q and R are inconsistent prove that $\leftarrow R$ is a conclusion from P and Q.

Part – B

1. a. What is meant by Tautology? Without truth table, show that
$((P \vee Q) \wedge \leftarrow (\leftarrow P \wedge (\leftarrow Q \vee \leftarrow R))) \vee (\leftarrow P \wedge \leftarrow Q) \vee (\leftarrow P \wedge \leftarrow R)$ is a tautology. (8)
   b. Show that the expression $(P \wedge Q) \vee (\leftarrow P \wedge Q) \vee (P \wedge \leftarrow Q) \vee (\leftarrow P \wedge \leftarrow Q)$ is a tautology by using truth tables. (8)
2. a. Show that $(\leftarrow P \wedge (\leftarrow Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ using truth table. Also prove this result without using truth table. (8)
   b. Without using truth tables, show that $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a tautology. (8)
3. a. Prove that $(P \rightarrow Q) \wedge (Q \rightarrow R)\_(P \rightarrow R)$. (8)
   b. Prove that $(\leftarrow Q \rightarrow \leftarrow P) \wedge (\leftarrow R \rightarrow \leftarrow Q)\_(P \rightarrow R)$. (8)
4. a. Obtain the DNF and CNF for $(P \rightarrow (Q \wedge R)) \wedge (\leftarrow P \rightarrow (\leftarrow Q \wedge \leftarrow R))$ (8)
   b. Obtain the principal disjunctive and conjunctive normal forms of

$(P \wedge Q) \vee (\leftarrow P \wedge R) \vee (Q \wedge R)$. (8)

5. a. Obtain the principal normal form of $(P \wedge Q) \vee (\leftarrow P \wedge R)$ (8)

   b. Obtain the PDNF for $(P \wedge Q) \vee (\sim P \wedge Q) \vee (Q \wedge R)$. (8)

6. a. Obtain the principal disjunctive normal form of $(P \wedge Q) \vee (\leftarrow P \wedge R)$
   (1) Using truth table.
   (2) Without using truth table. (8)

   b. Obtain the principal conjunctive normal form of the formula
   $(\leftarrow P \rightarrow R) \wedge (Q \leftrightarrow P)$.
   (i) Using truth table.
   (ii) Without using truth table (8)

7. a. Find the principal conjunctive and principal disjunctive normal forms of the formula $S \Leftrightarrow (P \rightarrow (Q \wedge R)) \wedge (\leftarrow P \rightarrow (\leftarrow Q \wedge \leftarrow R))$ (8)

   b. Obtain the product of sums canonical form for
   $(P \wedge Q \wedge R) \vee (\leftarrow P \wedge Q \wedge R) \vee (\leftarrow P \wedge \leftarrow Q \wedge \leftarrow R)$ (8)

8. a. Obtain pcnf and pdnf of the formula $(\leftarrow P \vee \leftarrow Q) \rightarrow (P \leftrightarrow \leftarrow Q)$, (8)

   b. Find the principal disjunctive and conjunctive normal forms of the formula $S \Leftrightarrow ((\leftarrow Q \vee \leftarrow R) \rightarrow \leftarrow P) \wedge (Q \vee R) \rightarrow P$. (8)

9. a. Obtain the DNF and CNF for $(P \rightarrow (Q \wedge R)) \wedge (\leftarrow P \rightarrow (\leftarrow Q \wedge \leftarrow R))$ (8)

   b. Obtain the principal disjunctive and conjunctive normal forms of
   $(P \wedge Q) \vee (\leftarrow P \wedge R) \vee (Q \wedge R)$. (8)

10. a. Obtain the principal normal form of $(P \wedge Q) \vee (\leftarrow P \wedge R)$ (8)

    b. Obtain the PDNF for $(P \wedge Q) \vee (\sim P \wedge Q) \vee (Q \wedge R)$. (8)

11. a. Obtain the principal disjunctive normal form of $(P \wedge Q) \vee (\leftarrow P \wedge R)$
    (1) Using truth table.
    (2) Without using truth table. (8)

    b. Using derivation process prove that $S \rightarrow \leftarrow Q$, $S \vee R, \leftarrow R$, $(\leftarrow R \leftrightarrow Q)\_\leftarrow P$.

12. a. Obtain the principal conjunctive normal form of the formula
    $(\leftarrow P \rightarrow R) \wedge (Q \leftrightarrow P)$.
    (i) Using truth table.
    (ii) Without using truth table (8)

    b. Find the principal conjunctive and principal disjunctive normal forms of the formula $S \Leftrightarrow (P \rightarrow (Q \wedge R)) \wedge (\leftarrow P \rightarrow (\leftarrow Q \wedge \leftarrow R))$ (8)

13. a. Obtain the product of sums canonical form for
    $(P \wedge Q \wedge R) \vee (\leftarrow P \wedge Q \wedge R) \vee (\leftarrow P \wedge \leftarrow Q \wedge \leftarrow R)$ (8)

    b. Obtain pcnf and pdnf of the formula $(\leftarrow P \vee \leftarrow Q) \rightarrow (P \leftrightarrow \leftarrow Q)$, (8)

14. a. Find the principal disjunctive and conjunctive normal forms of the Formula $S \Leftrightarrow ((\leftarrow Q \vee \leftarrow R) \rightarrow \leftarrow P) \wedge (Q \vee R) \rightarrow P$. (8)

    b. Show that the following premises are inconsistent.

a. If Jack misses many classes through illness, then he fails high school.
b. If jack fails high school, then he is uneducated.
c. If Jack reads a lot of books, then he is not uneducated.
d. Jack misses many classes through illness and reads a lot of books.(8)

15. a. Show that the following sets of premises are inconsistent
$P \to Q, P \to R, Q \to \leftarrow R, P$. (8)
b. Show that S is valid inference from the premises
$P \to \sim Q, Q \vee R, \sim S \to P$ and ~R (8)

16. a. Show that the following implication by using indirect method
$R \to \leftarrow Q, R \vee S, S \to \leftarrow Q, P \to Q \_ \leftarrow P$. (8)
b. Show that $J \wedge S$ logically follows from the premises
$P \to Q, P \to \leftarrow R, R, P \vee (J \wedge S)$. (8)

17. a. Show that the following sets of premises are inconsistent
$P \to Q, P \to R, Q \to \leftarrow R, P$. (8)
b. Show that $R \vee S$ is a valid conclusion from the premises
$C \vee D, (C \vee D) \to \sim H, \sim H \to (A \wedge \sim B)$ and $(A \wedge \sim B) \to (R \vee S)$ (8)

18. a. Show that S is valid inference from the premises
$P \to \sim Q, Q \vee R, \sim S \to P and \sim R$ (8)
b. Using conditional proof prove that $\leftarrow P \vee Q, \leftarrow Q \vee R, R \to S \_ P \to S$. (8)

19. a. Test the validity of the following argument. If I study, then I will pass in the examination If I watch TV, then I will not study. I failed in the examination.Therefore I watched TV. (8)
b. Let p,q,r be the following statement:
p: I will study discrete mathematics.
q: I will watch T.V.
r: I am in a good mood.
Write the following statements in terms of p,q,r and logical connectives.
a. If I do not study discrete mathematics and I watch T.V., then I am in a good mood..
b. If I am in a good mood, then I will study discrete mathematics or I will watch T.V.
c. If I am not in a good mood, then I will not watch T.V. or I will study discrete mathematics.
d. I will watch T.V. and I will not study discrete mathematics if and only if I am in a good mood. (8)

20. a. If there was rain, then traveling was difficult. If they had umbrella, then traveling was not difficult. They had umbrella. Therefore, there was no rain. Show that these statements constitute. (8)
b. By using truth table verify whether the following specifications are consistent: “Whenever the system software is being upgraded users can not access the file system. If users can access the file system, then they can save new files. If users can not save new files then the system software is not being upgraded”. (8)

21. a. Test the validity of the following argument. If I study, then I will pass in the examination. If I watch TV, then I will nor study. I failed in the

examination. Therefore I watched TV. (8)

b. Verify the validity of the inference. If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than Peter. Therefore, John is 'not successful than Peter. (8)

22. a. Without constructing the truth tables show that $A \vee C$ is not a valid consequence of the premises. $A \leftrightarrow (B \rightarrow C)$, $B \leftrightarrow (\leftarrow A \vee \leftarrow C)$, $C \leftrightarrow (A \vee \leftarrow B)$, B (8)

b. Using derivation process prove that $S \rightarrow \neg Q$, $S \vee R$, $\leftarrow R$, $(\leftarrow R \leftrightarrow Q)$_$\leftarrow P$.

UNIT – II

PREDICATE CALCULUS

PART – A (2 Marks)

1. Let P(x) denote the statement "x>4". What are the truth values of P(5) and P(2)?
2. Let Q(x,y) denote the statement " x= y +2 ", what are the truth values of the propositions Q(1,2) and Q(2,0).
3. What are the truth values of the propositions R(1,2,3) and R(0,01) ?
4. Express the statement "Good food is not cheep" in symbolic form.
5. Symbolize the statement "Given any positive integer, there is a greater positive integer.
6. If S = { -2,-1,0,1,2}, determine the truth value of , $\forall x \in S, |x|^2 \leq 3|x| - 2$
7. Give an example to show that $(\exists x)(A(x) \wedge B(x))$ need not be a conclusion Form $(\exists x)A(x)$ and $(\exists x)B(x)$.
8. where P:2 >1, Q(x) : x > 3 , R(x) : X > 4, with the universe of discourse E being E = { 2,3,4 }.
9. Express the statement " x is the father of the mother of y " in symbolic form.
10. Define simple statement function.
11. Express the statement " For every 'x' there exist a 'y' such that $x^2 + y^2 \geq 100$ ".
12. Define a compound statement function.
13. Define Quantifiers.
14. Define the term Universal Quantifier.
15. Define the term Existential quantifier.
16. Give the symbolic form of the statement "every book with a blue cover is a mathematics book".
17. Rewrite the following using quantifiers. " Some men are genius".
18. Write the following statement in the symbolic form " every one who likes fun will enjoy each of these plays".
19. Symbolize the expression " All the world lovers a lover"
20. Explain the existential and universal quantifier with an example each.

Part – B

1. a. Show that $(x)(P(x) \vee Q(x))$_$(x)P(x) \vee (\exists x)Q(x)$ by using indirect method.

(8)
b. Show that following implication

$(x)(P(x)\vee Q(x)), (x)(R(x)\rightarrow\leftarrow Q(x))\_(x)(R(x)\rightarrow\leftarrow P(x))$ (8)

2. a. Explain the two types of quantifiers and determine the truth table of each of the following statement:

(1) $\forall x, |x| = -x$

(2) $\forall x, x+2>x$

(3) $\exists x\ x^4 = x$,

(4) $\exists x, x-2=x$ (8)

b. Show that $(\exists x)M(x)$ follows logically from the premises

$(x)(H(x)\rightarrow M(x))$ and $(\exists x)H(x)$ (8)

3. a. Prove that $(\exists x)P(x)\rightarrow(x)Q(x)(x) \Rightarrow (P(x)\rightarrow Q(x))$ (8)

b. Use conditional proof to prove that $(x)(P(x)\rightarrow Q(x))\Rightarrow(x)P(x)\rightarrow(x)Q(x)$.(8)

4. a. Prove that $(\exists)(A(x)\vee B(x))\Leftrightarrow(\exists x)A(x)\vee(\exists x)B(x)$ (8)

b. Show that following implication

$(x)(P(x)\rightarrow Q(x)), (x)(R(x)\rightarrow\leftarrow Q(x))\_(x)(R(x)\rightarrow\leftarrow P(x))$ (8)

5. a. Show that $(x)(P(x)\rightarrow Q(x))\wedge(x)(Q(x)\rightarrow R(x))\_(x)(P(x)\rightarrow R(x))$ (8)

b. Is the following conclusion validly derivable from the premises given ?

If $(\forall x)(P(x)\rightarrow Q(x)), (\exists y)P(y)$ then $(\exists z)Q(z)$ (8)

6. a. Use indirect method of proof to show that

$(\forall x)(P(x)\vee Q(x))\_(\forall x)P(x)\vee(\exists x)Q(x)$ (8)

b. Prove that $(\exists x)P(x)\rightarrow(x)Q(x)\_(\forall x)(P(x)\rightarrow Q(x))$ (8)

7. a. Prove that $(\exists x)(A(x)\vee B(x))\Leftrightarrow(\exists x)A(x)\vee(\exists x)B(x)$ (8)

b. Explain the two types of quantifiers and determine the truth value of each of the following statement.. Show that

(i) $(\exists x)(F(x)\wedge S(x))\rightarrow(\forall y)M(y)\rightarrow W(y)$

(ii) $(\exists y)(M(y)\wedge\leftarrow w(y))$ the conclusion $(\exists x)[F(x)\rightarrow\leftarrow S(x)]$ follows. (8)

8. a. Prove that $(\exists x)M(x)$ follows logically from the premises $(x)(H(x)\rightarrow M(x))$ and $(\exists x)H(x)$. (8)

b. Prove that $(\exists x)A(x)\rightarrow B \Leftrightarrow (x)(A(x)\rightarrow B)$. (8)

9. a. Prove that

$(x)(H(x)\rightarrow A(x))\_(x)((\exists y)(H(y)\wedge N(x,y)))\rightarrow(\exists y)(A(y)\wedge N(x,y))$ (8)

b. Prove that $(\exists x)(P(x)\wedge Q(x))\_(\exists x)P(x)\wedge(\exists x)Q(x)$ (8)

10. a. Using CP or otherwise obtain the following implication.

$(\forall x)(P(x)\rightarrow Q(x)), (\forall x)(R(x)\rightarrow\leftarrow Q(x))\_(\forall x)(R(x)\rightarrow\leftarrow P(x))$. (8)

b. Verify the validity of the following argument:
Lions are dangerous animals. There are lions. Therefore there are dangerous animals. (8)

11. a. Show that from

(a) $(\exists x)(F(x)\wedge S(x))\rightarrow(y)(M(y)\rightarrow W(y))$

(b) $(\exists y)(M(y)\wedge \leftarrow W(y))$ the conclusion $(x)(F(x)\rightarrow\leftarrow S(x))$ follows. (8)

b. Prove that $(\exists x)A(x)\rightarrow B \Leftrightarrow (x)(A(x)\rightarrow B)$. (8)

12. a. (i)Give an example in which $(\exists x)(P(x)\rightarrow Q(x))$ is true but $((\exists x)P(x))\rightarrow((\exists x)Q(x))$ is false.

(ii) Find the truth value of $(x)(P\rightarrow Q(x))\vee(\exists x)R(x)$ (8)

b. (i) Let the Universe of discourse be E = {5,6,7}. Let A = { 5,6} and B = {6,7}. Let P(x) : x is in A; Q(x) : x is in B and R(x,y) : x+y < 12.

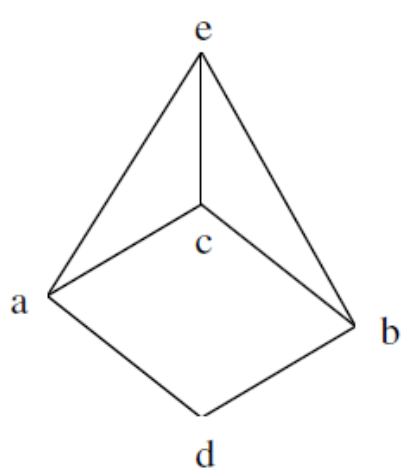
(ii) Find the truth value of $((\exists x)(P(x)\rightarrow Q(x))\rightarrow R(5,6)$. (8)

UNIT –III

SET THEORY

Part – A (2 Marks)

1. Given an example of a relation which is symmetric, transitive but not reflexive on $\{a,b,c\}$
2. Define partially ordered set.
3. The following is the Hasse diagram of a partially ordered set. Verify whether



a Lattics.

4. For any sets, A,B and C , prove that $AX(B\cap C)=(AXB)\cap(AXC)$.
5. Draw the Hasse-diagram of the set of partitions of 5.
6. Consider $(D_4,\leq)$ and $(D_9,\leq)$ where for positive integer n ,D_n denotes the set of all positive divisors of n and < is the divisibility. Obtain the Hasse diagram of the partially ordered set $L=D_4 XD_9$ under the product partial order.
7. Prove that every distributive lattice is modular but not conversely.

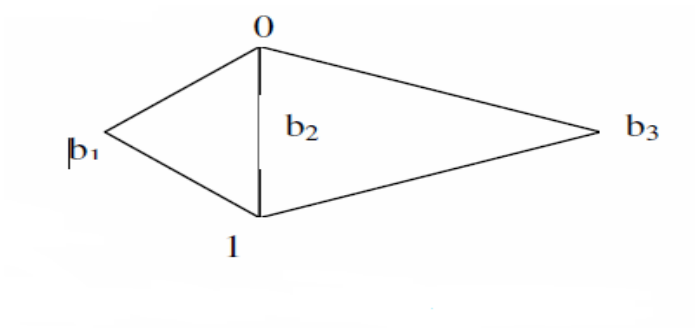
8. Give an example of a lattice which is modular but not a distributive.

9. Obtain the Hasse diagram of $(P(A_3), \subseteq)$ where $A_3 = \{a,b,c\}$

10. If $A = \{2,3\} \subseteq X = \{2,3,6,12,24,36\}$ and the relation $\leq$ is such that $x \leq y$ is x divides y, find the least element and greatest element for A.

11. Draw the Hasse diagram of $(X, \leq)$, where X is the set of positive divisors of 45 and the relation $\leq$ is such that $\leq \{(x, y : x \in A, y \in A \wedge (x \text{ divides } y)\}$

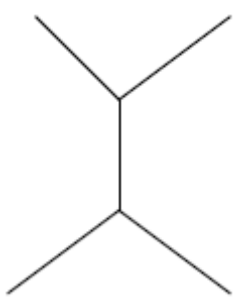
12. Show that the lattice.



Is not distributive.

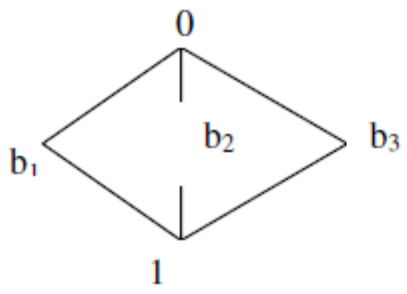
13 Is the lattice of divisors of 32 a Boolean algebra?.

14. Obtain the partial ordering represented by the Hasse diagram



15. If $R = \{(1,1), (1,2), (1,3)\}$ and $S = \{(2,1), (2,2), (3,2)\}$ are relations on the set $A = \{1,2,3\}$, verify whether R o $S = S$ o R by finding the relation matrices of R o S and S o R .

16. In the following lattice find $(b_1 \oplus b_3) * b_2$:



17. If a poset has a least element, then prove it is unique.
18. Partition $A = \{0,1,2,3,4,5\}$ with minsets generated by $B_1 = \{0,2,4\}$ and $B_2 = \{1,5\}$
19. Draw the Hasse diagram of $D_{20} = \{1,2,4,5,10,20\}$

Part – B

1. a. A survey of 500 television watchers produced the following information: 285 watch foot ball games; 195 watch hockey games; 115 watch basket ball games; 45 watch foot ball and basket ball games; 70 watch foot ball and hockey games; 50 watch hockey and basket ball games; 50 do not watch any of the three games; How many people watch exactly one of the three games. (8)

   b. In a Boolean algebra L prove that $(a \wedge b)' = a' \vee b'$ for all $a, b \in L$. (8)
2. a. Let the relation R be define on the set of all real numbers by 'if x,y are real numbers, $xRy \Leftrightarrow x$ - y is a rational numbers'. Show that R is an equivalence relation. (8)

   b. If R is an equivalence relation on a set A, prove that $[x] = [y]$ if only if xRy where $[x]$ and $[y]$ denote equivalence classes containing x and y respectively. (8)
3. a. In a lattice show that $a \le b \Rightarrow a * b = a$. (8)

   b. Prove that every chain is a distributive lattice. (8)
4. a. If D_{45} denotes the set of all divisors of 45 under divisibility ordering, find which elements have complements and which do not have complements.(8)

   b. In any Boolean algebra show that $a = 0 \Leftrightarrow ab' + a'b = b$. (8)
5. a. Draw the Hasse Diagram of the Lattice L of all subsets of { a, b , c } under intersection and union. (8)

   b. Show that every totally ordered set is a lattice. (8)
6. a. Show that in a lattice both distributive inequalities and isotone property are true. (8)

b. Show that in a Boolean algebra $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$. (8)

7. a. Prove that distinct equivalence classes are disjoint. (8)
   b. In a Lattice show that $a \le b$ and $c \le d$ implies $a * b \le b * d$. (8)
8. a. Let P = {{1,2},{3,4},{5}} be a partition of the set S = { 1,2,3,4,5}. Construct an equivalence relation R on S so that equivalence classes with respect to R are precisely the members of P. (8)
   b. Show that a chain with three or more elements is not complemented.(8)
9. a. Establish De Morgan's Laws in a Boolean Algebra. (8)
   b. In a distributive lattice prove that $a * b = a * c$ and $a \oplus b = a \oplus c$ implies that $a * b \le b * d$. (8)
10. a. Define the relation P on $\{1,2,3,4,5\}$ by $P = \{(a,b) / a \square b = 1\}$ determine theadjacency matrix of $_2P$ (8)
    b. Let $(L, \le)$ be a lattice. For $a, b, c \in L$ if $b \le c$, prove that $a * b \le a * c$ and $a \oplus b \le a \oplus c$. (8)
11. a. In a distributive lattice, show that
    $(a * b) \oplus (b * c) \oplus (c * a) \le (a \oplus b) * (b \oplus c) * (c \oplus a)$. (8)
    b. If γ_1, γ_2 are equivalence relations in a set A , then prove $\gamma_1 \cap \gamma_2$ is anequivalence relation in A . (8)
12. a. Simplify the Boolean expression $((x_1 + x_2) + (x_1 + x_2)).x_1.\overline{x_2}$ (8)
    b. State and prove the distributive inequalities of a lattice. (8)

UNIT – IV

FUNCTIONS

Part – A (2 Marks)

1. List all possible functions from X = { a,b,c} to Y = {0,1} and indicate in each case whether the function is one-to-one is onto and is one-to-one and onto.
2. If A = { 1,2,3.....n} show that any function from A to B, which is one-toone must also be onto and conversely.
3. If A has m elements and **B** has n elements, how many functions are there from A to **B** .
4. Let $f : R \to R$ and $g : R \to R$ where R is the set of real numbers find f o g and g o f , if $f(x) = x^2 - 2$ and $g(x) = x + 4$.
5. Show that the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R$, are inverse of one another.
6. Let f , g,h be functions from N to N where N is the set of natural numbers so that $f(n) = n + 1$, $g(n) = 2n$, $h(n) = 1$, n is even and = 0, n is odd. Determine f o f , f o g, g o f and (f o g) o h .
7. The inverse of the inverse of a function is the function itself i.e. $(f^{-1})^{-1} = f$,

   (OR)

   If a function g be the inverse of a function f then f is the of g .

8. Show that $x * y = x - y$ is not a binary operation over the set of natural numbers, but that it is a binary operation on the set of integers. Is it commutative or associative?

9. Determine whether usual multiplication on the set $A = \{1, -1\}$ is a binary operation.

10. Examine whether matrix multiplication on the set M = $\left\{ \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in R \right\}$ is binary operation .

11. What are the identity and inverse elements under $*$ defined by $a * b = \frac{ab}{2}, \forall a, b \in R$

12. What do mean by a primitive recursive function?

13. Show that the function $f(x, y) = x + y$ is primitive recursive.

14. Show that $f(x, y) = x^Y$ is primitive function.

15. Let $\left[\sqrt{x} \right]$ be the greatest integer less than or equal to $\sqrt{x}$. Show that $\left[\sqrt{x} \right]$ is primitive recursive.

16. Show that set of divisors of a positive integers n is recursive.

17. Is the permutation p= $\begin{pmatrix} 2 & 4 & 5 & 7 & 6 & 3 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$ even or odd?

17. If A has 3 elements and B has 2 elements, how many functions are there from A to B.

18. Define characteristic function.

19. If $f : A \to B$ and $g : B \to C$ are mappings and g o $f : A \to C$ is one-to one (Injection), prove that f is one-to-one.

20. If ψ_A denotes the characteristic function of the set A , prove that $\psi_{A \cup B}(x) = \psi_A(x) + \psi_B(x) - \psi_{A \cap B}(x)$ for all $x \in E$, the universal set.

21. Find all the mapping from A = { 1,2 } to B = { 3,4 }.

22. Let $h(x, y) = g\left(f_1(x, y), f_2(x, y) \right)$ for all positive integers x and y, where $f_1(x, y) = x^2 + y^2$, $f_2(x, y) = x$ and $g(x, y) = x y^2$. Find $h(x, y)$ in terms of x and y.

23. If $f(x, y) = x + y$,express $f(x, y + 1)$ in term of successor and projection functions.

24. Give an example of a commutative ring without identity.

Part – B

1. a. Let the function f and g be defined and $g(x) = x^2 - 2$. Determine the composition function f o g and g o f . (8)

   b. Let a and b be positive integers and suppose Q is defined recursively as follows: $Q(a,b) = 0$, if a<b $= Q(a - b, b) + 1$ if b≤a .Find Q(2,5), Q(12,5), Q(5861,1) (8)

2. a. Let $f : R \to R$ be defined by $f(x) = 2x - 3$. Find a formula for f^{-1}. (8)

b. Show that $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ by using characteristic function. (8)

3. a. Let Z^+ denote the set of positive integers and Z denote the set of integers.

Let $f: Z^+ \to Z$: be defined by $f(n)=\begin{cases}\frac{n}{2}, & if\ n\,is\,even\\ \frac{1-n}{2}, & if\ n\,is\,odd\end{cases}$

Prove that f is a bijective. (8)

b. Let A,B,C be any three nonempty sets. Let $f: A\to B$ and $g: B\to C$ be mappings. If f and g are onto, prove that g o $f: A\to C$ is onto. Also give an example to show that g o f may be onto but both f and g need not be onto.(8)

4. a. If R denotes the set of real numbers and $f: R\to R$ is given by $f(x)=x^3-2$, find f^{-1}. (8)

b. If f = $\begin{pmatrix}1&2&3&4\\3&2&1&4\end{pmatrix}$ and g=$\begin{pmatrix}1&2&3&4\\2&3&4&1\end{pmatrix}$ are

permutations, prove that $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$(8)

5. a. Find all mappings from A = { 1 ,2, 3 } to B = { 4,5 }; find which of them are one-to-one and which are onto. (8)

b. If f and g are bijective on a set A , prove that f o g is also a bijection.(8)

6. a. If f = $\begin{pmatrix}1&2&3&4&5\\2&4&5&1&3\end{pmatrix}$ and h=$\begin{pmatrix}1&2&3&4&5\\5&2&4&3&1\end{pmatrix}$ are

permutations on the set A = { 1, 2,3, 4 ,5 } find a permutation g on A such that f o $g=h$ o f .(8)

b. The Ackerman function $A(x, y)$ is defined by $A(x, y)=y+1$; $A(x+1,0)=A(x,1)$; $A(x+1, y+1)=A(x, A(x+1, y))$. Find $A(2,1)$. (8)

7. a. Let $a<b$. If $f: [a,b]\to[0,1]$ is defined by $f(x)=\frac{x-a}{a-b}$, prove

that f is a bijection and find it inverse (Here $[a,b]$ and $[0,1]$ are closed intervals). (8)

b. If $f: A\to B$ and $g: B\to C$ are mappings such that g o $f: A\to C$ is onto, prove that g is onto. Also give an example in which g o f is onto while f is not onto.(8)

8. a. Let I be the set of integers, I_+ be the set of positive integers, and

$I_P=\{0,1,,....p-1\}$. Determine which of the following function are one to one, which are onto, and which are ono-to-one onto.

(i) $f: I\to I$ defined by $f(j)=\begin{cases}\frac{j}{2}, & if\ j\,is\,even\\ \frac{j-1}{2}, & if\ j\,is\,odd\end{cases}$

(ii)$f: I_+\to I_+$ define by $f(x)=$ greater integer $\le\sqrt{x}$

(iii) $f : I_7 \to I_7$ defined by $f(x) = 3x(\text{mod}7)$.

(iv) $f : I_4 \to I_4$ defined by $f(x) = 3x(\text{mod } 4)$. (8)

b. Let $f : X \to Y$ and $f : Y \to X$. Prove that the function g is equal to f^{1} only if $g \text{ o } f = I_x$ and $f \text{ o } g = I_y$ in the usual notation. (8)

9. a. Let $X = \{1,2,3\}$ and f , g,h and s be functions from X to X given by

$f = \{(1,2), (2,3), (3,1)\}$

$g = \{(1,2), (2,1), (3,3)\}$

$h = \{(1,1), (2,2), (3,1)\}$

$s = \{(1,1), (2,2), (3,3)\}$

Find f o g, g o f , f o g o f , s o g, g o s, s o s *and* f o s. (8)

b. Let $f : X \to Y$ and $g : Y \to Z$ and both f and g are onto, show that g o f is also onto. Is g o f is one-to-one if both g and f are one-toone? (8)

10. a. How many functions are there from X to Y for the sets given below? Find also the number of functions which are one-one onto and bijective.

(a) $X = \{a,b,c\}$ $Y = \{a,b,c\}$

(b) $X = \{a,b,c,d\}$ $Y = \{a,b,c\}$

(c) $X = \{a,b,c\}$ $Y = \{a,b,c,d\}$ (8)

b. The composite of two one-to-one and onto functions is also a one-to one and onto function. OR) The composite of two bijections is also a bijection.(8)

11. a. Let $f : X \to Y$ and $g : Y \to Z$ be any two invertible functions, then $f : X \to Y$ and $(g \text{ o } f)^{-1} . = f^{-1} \text{ o } g^{-1}$ (8)

b. Using characteristic function show that Let $\overline{A \cup B} = (\overline{A}) \cap (\overline{B})$ (8)

12. a. Show that if $f(x, y)$ defines the remainder upon division of y by x , then it is primitive recursive function. (8)

b. Show that the function $f(x) = x!$ is primitive recursive, where 0! = 1 and $n! = nx(n - 1)!$. (8)

UNIT-5

GROUPS

Part – A (2 Marks)

1. Define when an algebraic system $S,+,.$
2. Define semigroup and monoid. Give an example of a semigroup which is not a monoid.
3. Give an example of a monoid which is not a group.
4. Define semi group homomorphism.
5. When will a set of element from a monoid?
6. Given an example of sub semi-group.

7. Prove that the only idempotent element of a group is its identity element.
8. Prove that monoid homomorphism preserves invertibility and monoid epimorphism preserves zero element (if it exists).
9. What do you call a homomorphism of a semigroup into itself?
10. Consider the set N of positive integers, and * be the operation of least common multiple (l.c.m) on N.
11. Find the left cosets of {[0],[3]} in the addition modular group (Z_6 , $+_6$).
12. If H is a subgroup of the group G, among the right cosets of H in G, prove that there is only one subgroup viz., H.
13. If H is a subgroup of the group G, among the right cosets of H in G, prove that there is only one subgroup viz., H.
14. Consider the group G = { 1,2,3,4,5,6}multiplication modulo 7. Find the multiplication table of G.]
15. State Lagrange's theorem for finite groups. Is the converse true?
16. Define ring and given an example of an ring with zero-divisors.
17. When an element is a ring is said to be zero divisor? Give an example of a ring without zero divisors ?
18. Show that if every element in a group is its own inverse, then the group must be abelian.
19. Show that (Z_5, $+_5$) is a cyclic group.
20. Find the all cosets of the sub group H = {1, -1 } in G = { 1 -1 , i , -i } with the operation multiplication.
21. Show that every cyclic group of order n is isomorphism to the group (Z_n, $+_n$)
22. If f is a homomorphism of a group G into a group G' then prove that group n homomorphism preserves identities
23. Define normal subgroup of a group.
24. Prove that a subgroup $S(\neq,\phi)$ of a group (G,*) is a sub group if and only if for any pair of elements $a, b \in S$,$a * b^{-1} \in S$
25. Define Integral Domain. Given example of a finite integral domain which is also a field.
26. Define a field.
27. Define when an algebraic system(S,+,.) is called a ring.
28. Define Group Code and encoding function.

Part – B

1. a. Show that group homomorphism preserves, identity, inverse and Subgroup (8)

b. Obtain all the elements of (S_3, Δ) and also construct the composition table with respect to the operation Δ).. (8)

2. a. Prove that monoid homomorphism preserves invertibility and monoid epimorphism preserves zero element (if it exists). (8)

b. For any commutative monoid (M,*) the set of idempotent element of M forms a sub monoid. (8)

3. a. Show that the set N of natural numbers is a semigroup under the operation x*y = max { x , y } . Is it a monoid? (8)

b. Prove that a subset S($\neq,\phi$) is a sub group if and only if for pair of element $a, b \in S$,$a * b^{-1} \in S$, (8)

4. a. Show that the mapping g from the algebraic system (S,+) to the system (T,X) define by g(a) = 3^a , where S is the set all rational numbers under addition + and T is the set of non-zero real numbers under multiplication operation X, is a homomorphism but not an isomorphism. (8)

b. Find all the non-trivial subgroups of (Z_6, $+_6$).8)

5. a. The intersection of any two subgroups of a group G is again a subgroup of G. – Prove. (8)

b. Show that in a finite group, order of any subgroup divides the order of the group. (8)

6. a. Let $f : (G,*) \rightarrow (H,\Delta)$ be group homomorphism. Then show that Ker(f) is a normal subgroup. (8)

b. State and prove Lagrange's theorem for finite group. (8)

7. a. Let S be a non-empty set and P(S) denote the power set of S. Verify whether $(P(S),\cap)$ is a group. (8)

b. If $_1H$ and $_2H$ are sub-groups of a group (G,*), prove that $H_1 \cap H_2$ is a sub-group of G. (8)

8. a. If G_1 and G_2 are groups and $f : G_1 \rightarrow G_2$ is a homomorphism, prove that the kernel of f viz, ker f is a normal sub-group of G_1 . (8)

b. Prove that every finite group of order n is isomorphic to a permutation group of degree n. (OR Cayley's Thereem) (8)

9. a. Prove that the direct product of two groups is a group. (8)

b. Let (G, *) be a finite cyclic group generated by an element $a \in G$. If G is of order n, prove that $a^n = e =$ and $G \left\{ a, a^2, \ldots\ldots a^n = e \right\}$ where n is least positive integer for which $a^n = e$ (8)

10. a. Let G be group and $a \in G$. Let $f : G \rightarrow G$ be given by $f(x) = axa^{-1}$ for all $x \in G$. Prove that f is an isomorphism on G on to G. (8)

b. Obtain all the elements of S_3 . Construct the composition table of S_3 with respect to the operation Δ . Is (S_3 , Δ) is abelian? Justify your answer. (8)

11. a. If (G, *) is an abelian group and if for all $a,b \in G$. Show that $(a * b)^n = a^n * b^n$ for every integer n. (8)

b. Show that the set of all idempotent elements of a commutative monoid (M, *) forms a submonoid of (M, *). (or) Show that every subgroup of a cyclic group is cyclic. (8)

12. a. Let $(A,*)$ be a group. Let H = { a / $a \in G$ and a*b = b * a $\forall b \in G$ }. Show that H is a normal subgroup. (8)

b. Define a cyclic group. Prove that any group of prime order is cycle. (8)

13. a. Let (H, .) be a subgroup of (G,.) . Let N ={x / $x \in G$, $xHx^{-1} = H$, } show that (N, .) is a subgroup is G. (8)

b. If (G, *) is a finite group of order n, then for and $a \in G$, we must have $a^n = e$ were e is the identity of the group G. (8)

14. a. How many generators are there in a cyclic group of order 10 ? (8)

b. Let (H, *) be a subgroup of G,* . Then show that (H, *) is a normal subgroup iff $a * h * a^{-1} = H,\ \forall a \in G$ (8)

15. a. Prove that the kernel of a homomorphism g from the group (G, *) to another group (H,Δ) is a normal subgroup (G,*) (8)

b. Show that (2,5) encoding function defined by e(00) = 00000, e(01)=01110, e(10) = 10101, e(11) = 11011 is a group code. (8)

16. a. Find the minimum distance of the encoding function $e : B^2 \to B^4$ given by e(00)=0000 ,e(10)=0110,e(01) = 1011, e(11) = 1100. (8)

b. If H= $\begin{bmatrix} 1100001 \\ 1010010 \\ 1010100 \\ 0111000 \end{bmatrix}$ is the parity check matrix, find the Hamming code generated by H (in which the first three bits represent information portion and the next four bits are parity check bits). If y = (0, 1, 1, 1, 1, 1, 0) is the received word find the corresponding transmitted code word. (8)

17. a. Let H= $\begin{bmatrix} 011001 \\ 101010 \\ 110100 \end{bmatrix}$ be a parity check matrix. Find

(i). The Hamming code generated byH.

(ii). Find the minimum distance of the code.

(iii). If 00110 is the received word find the corresponding transmitted code word. (8)

b.Show that (m, m+1) parity cheek code e :$B^m \to B^{m+1}$ is a group , (8)